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Meissner effect, diamagnetism, and classical physics—a review

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We review the literature on what classical physics says about the Meissner effect and the London equations. We discuss the relevance of the Bohr-van Leeuwen theorem for the perfect diamagnetism of superconductors and conclude that the theorem is based on invalid assumptions. We also point out results in the literature that show how magnetic flux expulsion from a sample cooled to superconductivity can be understood as an approach to the magnetostatic energy minimum. These results have been published several times but many textbooks on magnetism still claim that there is no classical diamagnetism, and virtually all books on superconductivity repeat Meissner’s 1933 statement that flux expulsion has no classical explanation. ©2012 American Association of Physics Teachers. [DOI: 10.1119/1.3662027]

I. INTRODUCTION

It is now a century since superconductivity was discovered by Kammerlingh-Onnes in Leiden in 1911. From the beginning, there was considerable interest from theoretical physicists. Unfortunately progress has been slow and one can safely say that the phenomenon is still not completely understood, at least not in a fundamental reductionist sense. It is therefore important that the things that can be understood are correctly presented in textbooks. To the contrary, textbooks often repeat two myths which have become so ingrained in the minds of physicists that they hamper progress. These are:

• There is no classical diamagnetism (Bohr,1 van Leeuwen2).
• There is no classical explanation of flux expulsion from a superconductor (Meissner3).

Although both statements have been disproved multiple times in the scientific literature, these myths continue to be spread. We hope this review will improve the situation.

Meissner found it natural that a weak magnetic field could not penetrate a Type-I superconductor. The fact that this is already in conflict with classical physics, according to the Bohr-van Leeuwen theorem is rarely mentioned. On the other hand, Meissner found it remarkable that a normal metal with a magnetic field inside will expel this field when cooled to superconductivity. This was claimed to have no classical explanation, and consequently superconductors were not considered perfect conductors.

We will not present any new results in this paper. Instead, we will review some of the strong evidence in the archival literature demonstrating that the above two statements are false. We begin with the Bohr-van Leeuwen theorem and then turn to the classical explanation of flux expulsion. After discussing the arguments for the traditional point of view, we will demonstrate why they are wrong.

II. THE BOHR-VAN LEEUWEN THEOREM AND CLASSICAL DIAMAGNETISM

The Hamiltonian $H$, for a system of charged particles interacting via a (scalar) potential energy $U$ is

$$H(r_j, p_j) = \sum_{j=1}^{N} \frac{p_j^2}{2m_j} + U(r_j).$$  \hspace{1cm} (1)

The particles have masses $m_j$, position vectors $r_j$, and momenta $p_j$. The presence of an external magnetic field with vector potential $A(r)$ alters the Hamiltonian to

$$H(r_j, p_j) = \sum_{j=1}^{N} \left[ \frac{p_j^2}{2m_j} - \frac{e_j}{2m_j} \left( A(r_j) \right)^2 \right] + U(r_j),$$  \hspace{1cm} (2)

where $e_j$ are the charges of masses $m_j$. Using this equation one can show that statistical mechanics predicts that the energy—the thermal average of the Hamiltonian—does not depend on the external field. Hence, the system exhibits neither a paramagnetic nor a diamagnetic response.

In his 1911 doctoral dissertation, Niels Bohr1 used the above equation and concluded there is no magnetic response of a metal according to classical physics. In 1919 Hendrika Johanna van Leeuwen independently came to the same conclusion in her Leiden thesis. When her work was published in 1921, Miss van Leeuwen2 noted that similar conclusions had been reached by Bohr.

Many books on magnetism refer to the above results as the Bohr-van Leeuwen theorem, or simply the van Leeuwen theorem. The theorem is often summarized as stating there is no classical magnetism. Since this is obvious in the case of spin or atomic angular momenta—the quantum phenomena responsible for paramagnetism—the more interesting conclusion is that classical statistical mechanics and electromagnetism cannot explain diamagnetism. A very thorough treatment of the Bohr-van Leeuwen theorem can be found in Van Vleck.4 Other books, such as Mohn,5 Getzlaff,6 Aharoni,7 and Lévy,8 mention the theorem more or less briefly. Weaknesses in classical derivations of diamagnetism in modern textbooks have been pointed out by O’Dell and Zia.9 A discussion of the theorem can also be found in the Feynman lectures10 (Vol. 2, Sec. 34-6), which points out that a constant external field will not do any work on a system of charges. Therefore, the energy of this system cannot depend on the external field. In addition, the fact that the diamagnetic response of most materials is very small lends empirical support to the theorem.

A. The inadequacy of the basic assumptions

If we accept the popular version of the Bohr-van Leeuwen theorem as true, then one must conclude that the perfect...
diamagnetism of superconductors cannot have a classical explanation. A study of the proof of the theorem, however, reveals that it is only valid under assumptions that do not necessarily hold. The assumed Hamiltonian of Eq. (2) only includes the vector potential of the external magnetic field. But it has been known since 1920 that the best Hamiltonian for a system of classical charged particles is the Darwin Hamiltonian. In this Hamiltonian one takes into account the internal magnetic fields generated by the moving charged particles of the system itself, plus any corresponding interactions.

The simplest way to see that the total energy of a system of charged particles depends on the external field is to note that the total energy includes a magnetic energy

\[ E_B = \frac{1}{8\pi} \int B^2(r) dV = \frac{1}{8\pi} \left( B_c + B_i \right)^2 dV, \tag{3} \]

where \( B_c \) is the external magnetic field and \( B_i \) is the internal field. Using the Biot-Savart law this field is given, to first order in \( v/c \), by

\[ B_i(r) = \sum_{j=1}^{N} B_j(r) = \sum_{j=1}^{N} \frac{e_j v_j \times (r - r_j)}{c |r - r_j|^3}. \tag{4} \]

Equation (3) makes it obvious that to minimize the energy the internal field will be, as much as possible, equal in magnitude and opposite in direction to the external field. This gives diamagnetism.

Charles Galton Darwin was first to derive an approximate Lagrangian for a system of charged particles (neglecting radiation) that is correct to order \( (v/c)^2 \). In a time independent external magnetic field, there is then a conserved Darwin energy given by

\[ E_D = \sum_{j=1}^{N} \left[ \frac{m_j}{2} v_j^2 + U(r_j) + \frac{1}{2} \mathbf{A}_e(r_j) \cdot \mathbf{v}_j + \mathbf{A}_i(r_j) \right] + U(r_k) + E_c. \tag{5} \]

In this equation, \( v_j \) are velocity vectors, \( \mathbf{A}_e \) is the external vector potential, \( \mathbf{A}_i \) is the (constant) energy of the external magnetic field, and \( \mathbf{A}_k \) is the internal vector potential given by

\[ \mathbf{A}_i(r_j; r_k, v_k) = \sum_{k \neq j} \frac{e_k v_k + (v_k \cdot \mathbf{e}_{kj}) \mathbf{e}_{kj}}{2c}, \tag{6} \]

where \( r_{kj} = |r_j - r_k| \) and \( \mathbf{e}_{kj} = (r_j - r_k)/r_{kj} \). Although there is a corresponding Hamiltonian, it cannot be written in closed form. When the Darwin magnetic interactions are taken into account, the Bohr-van Leeuwen theorem is no longer valid because the magnetic fields of the moving charges will contribute to the total magnetic energy. The fact that this invalidates the Bohr-van Leeuwen theorem for superconductors was stated explicitly by Pfleiderer in a letter to Nature in 1966. A more recent discussion of classical diamagnetism and the Darwin Hamiltonian is given by Essén.

The rotation of such a system produces a circulating current and thus a magnetic field. Simple calculations show that this field is of opposite direction to the external field and so the system is diamagnetic. This idea was first used by Langevin to derive diamagnetism. But from the point of view of the Bohr-van Leeuwen theorem this seems strange. In Feynman’s lectures, the Bohr-van Leeuwen theorem is therefore not considered to be valid for systems that can rotate. As noted above, the problem is simply that the Bohr-van Leeuwen theorem does not take into account the magnetic field produced by the particles of the system itself. When this internal field is accounted for, diamagnetism follows naturally, whether for systems of atoms from the Larmor’s theorem or for superconductors from the Darwin formalism.

C. The Shanghai experiment—measuring a diamagnetic current?

If it is true that the classical Hamiltonian for a system of charged particles predicts diamagnetism, then why is the phenomenon so weak and insignificant in most cases? Is there any evidence for classical diamagnetism for systems other than superconductors? As stressed by Mahajan, plasmas are typically diamagnetic. However, plasmas are not usually in thermal equilibrium so it is difficult to reach any definite conclusions from them. An experiment on an electron gas in thermal equilibrium, performed by Xinyong Fu and Zitao Fu in Shanghai, is therefore of considerable interest.

Two electrodes of Ag-O-Cs side-by-side in a vacuum tube emit electrons at room temperature because of their low work function. If a magnetic field is imposed on this system, an asymmetry arises and electrons flow from one electrode to the other. For a field strength of about 4 gauss a steady current of \( \sim 10^{-14} \) A is measured at room temperature. The current grows with increasing field strength and changes direction as the polarity is reversed. The authors interpret this result as if the magnetic field acts as a Maxwell demon that can violate the second law of thermodynamics.

In view of statistical mechanics based on the Darwin Hamiltonian, it is more natural to interpret this result as a diamagnetic response of the system. The current—just as the super-current of the Meissner effect—is due to a diamagnetic thermal equilibrium. This means that no useful work can be extracted from the system.

III. ON THE ALLEGED INCONSISTENCY OF MAGNETIC FLUX EXPULSION WITH CLASSICAL PHYSICS

Meissner and Ochsenfeld discovered the Meissner effect in 1933. To their surprise a magnetic field was not only unable to penetrate a superconductor, it was also expelled from the interior of a conductor as it was cooled below its critical temperature. The first effect—ideal diamagnetism—seemed natural to them even though it violates the Bohr-van Leeuwen theorem. As already discussed, by violating this theorem ideal diamagnetism should have been considered a non-classical effect. The flux expulsion, on the other hand, was explicitly proclaimed by Meissner to have no classical explanation. Meissner does not give any arguments or references to support this statement, but according to Dahl the theoretical basis was Lippmann’s theorem on the conservation of magnetic flux through an ideally conducting current loop. Later, Forrest and others have argued...
that the magnetohydrodynamic theorem on frozen-in flux lines also supports this notion. In this section, we discuss these arguments and point out that they do not rule out a classical explanation of the Meissner effect.

A. Lippmann’s theorem

Gabriel Lippmann (1845–1921), winner of the 1908 Nobel Prize in Physics, published a theorem in 1889 stating that the magnetic flux through an ideally conducting current loop is conserved. In 1919, when superconductivity had been discovered, Lippmann again published this result in three different French journals (see Sauer). Although the idea of flux conservation is considered highly fundamental, nowadays references to Lippmann’s theorem are hard to find. But at the time Lippmann’s theorem was quite influential, and Dahl explains how this was one of the results that made the Meissner effect seem surprising—and non-classical—at the time of its discovery.

The proof of Lippmann’s theorem follows by noting that the self inductance $L$ of a closed circuit, or loop, is related to the magnetic flux $\Phi$ from the current in the loop via

$$\Phi = L\dot{q},$$

(7)

where $c$ is the speed of light and $\dot{q}$ is the current through the circuit, an overdot denoting a time derivative (see Landau and Lifshitz, Vol. 8, §33). The equation of motion for a single loop electric circuit is

$$L\dot{q} + R\dot{q} + C^{-1}q = E(t),$$

(8)

where $R$ is the resistance, $C$ is the capacitance, and $E(t)$ is the emf driving the current. If there is no resistance, no capacitance, and no emf, this equation becomes

$$L\dot{q} = 0.$$  

(9)

Therefore, if the self inductance $L$ is constant, Eqs. (7) and (9) tell us that $\Phi = \text{constant}$.

B. Lippmann’s theorem and superconductors

Although Lippmann’s theorem is correct, its relevance for the prevention of flux expulsion is not clear. For superconductors there are two points to consider, the assumption of zero emf, and the fact that constant flux does not imply constant magnetic field. We consider these points one at a time.

Consider a superconducting sphere of radius $r$ in a constant external magnetic field $B_e$. When the sphere expels this field by generating (surface) currents that produce $B_s = -B_e$ in its interior, the total magnetic energy is reduced. The magnetic energy change is

$$\Delta E_B = -3 \left(\frac{4\pi}{3}\right) \frac{B_e^3}{8\pi},$$

(10)

or three times the initial interior magnetic energy. This energy is thus available for producing the emf required to generate currents in the sphere’s interior. The assumption of Lippmann’s theorem—that the emf is zero—is therefore not fulfilled.

When a steady current flows through a fixed metal wire both the flux and the magnetic field distribution are constant. On the other hand, when current flows in a loop in a conducting medium, a constant flux through the loop does not imply a constant magnetic field because the loop can change in size, shape, or location. Normally current loops are subject to forces that increase their radius (see e.g., Landau and Lifshitz, Vol. 8, §34, Problem 4, or Essén, Sec. 4.1). In view of this, Lippmann’s theorem does not automatically imply that the magnetic field must be constant, even if the emf is zero.

Other authors have reached similar conclusions. Mei and Liang carefully considered the electromagnetics of superconductors in 1991 and write, “Thus Meissner’s experiment should be viewed through its time history instead of as a strictly dc event. In that case classical electromagnetic theory will be consistent with the Meissner effect.”

C. Frozen-in field lines

The simplest derivation of the frozen-in field result for a conducting medium begins with Ohm’s law $j = \sigma E$. If $\sigma \to \infty$ we must have $E = 0$ to prevent infinite current. Faraday’s law then gives $\partial B / \partial t = -c(\nabla \times E) = 0$ which tells us the magnetic field $\mathbf{B}$ is constant (Forrest, Alfén and Fälthammar).

When dealing with a conducting fluid, the equations of magnetohydrodynamics and the limit of infinite conductivity in Ohm’s law give rise to

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}).$$

(11)

This result tells us the magnetic field convects with the fluid but does not dissipate. In addition, Eq. (11) can be used to derive the following two statements that are often referred to as the frozen-in field theorem:

1. the magnetic flux through any closed curve moving with the fluid is constant, and
2. a magnetic field line moving with the fluid remains a magnetic field line for all time.

The first of these is a just a restatement of Lippmann’s theorem for a current loop in a fluid.

D. Magnetic field lines in superconductors

As shown in the previous section, Ohm’s law with infinite conductivity implies there can be no change in the magnetic field since this would give rise to an infinite current density. It is, however, not physically meaningful to take the limit $\sigma \to \infty$ in Ohm’s law. In a medium of zero resistivity one must instead use the equation of motion for the charge carriers. One can then derive

$$\frac{dj}{dt} = \frac{e^2}{m} E + \frac{e}{mc} \mathbf{j} \times \mathbf{B},$$

(12)

for the time rate of change of the current density. The time derivative here is a convective, or material, time derivative. Ohm’s law simply does not apply, and therefore it cannot imply that the magnetic field does not change.

The second statement of the frozen-in field theorem can be questioned on the same grounds. Just as with Lippmann’s theorem, the conclusion that the magnetic field remains constant does not follow, even if the statement is assumed valid. The magnetic field is determined by the density of magnetic field lines and constancy of this density requires that the conducting fluid is incompressible. One can easily imagine that
the fluid of superconducting electrons is compressible, and that magnetic pressure pushes the fluid (with its magnetic field lines) to the surface of the material. Corroborating this point of view, Alfvén and Fälthammar (Sec. 5.4.2) state that “in low density plasmas the concept of frozen-in lines of force is questionable.”

E. A classical derivation of the London equations

In 1981 Edwards published a manuscript with the above title in Physical Review Letters. This caused an uproar of indignation and the journal later published three different criticisms of Edwards’ work. Incidentally, all of Edwards’ critics restated (or implied) the textbook myth that the Meissner flux expulsion does not have a classical explanation. In addition, the journal Nature published a study by Taylor pointing out the faults of Edwards’ derivation. Of course, Edwards is not the only one to publish an erroneous derivation of the London equations. A much earlier example is the derivation by Moore from 1976. The fact that various derivations have been wrong, of course, does not prove anything—as long as a correct derivation exists.

In 1966 Nature published a classical explanation of the London equations by Pfleiderer which did not cause any comment. Indeed it has not been cited a single time, probably because it is very brief and cryptic. However, a classical derivation of the London equations and flux expulsion can be found in a classic textbook by the French Nobel laureate Pierre Gilles de Gennes. Because this derivation should be more recognized, we repeat it here.

F. de Gennes’ derivation of flux expulsion

The total energy of the relevant electrons in the superconductor is assumed to have three contributions: the condensation energy associated with the phase transition $E_S$, the energy of the magnetic field $E_B$, and the kinetic energy of the moving superconducting electrons $E_k$. The total energy relevant to the problem is thus taken to be

$$E = E_S + E_B + E_k.$$ (13)

The condensation energy is then assumed to be constant while the remaining two can vary in response to external field variations. The super-current density is written as

$$j(r) = n(r)e\nu(r),$$ (14)

where $n$ is the number density of superconducting electrons and $\nu$ is their velocity, which gives a kinetic energy of

$$E_k = \frac{1}{2}n(r)mv^2(r)\,dV = \frac{1}{2}\frac{m}{2e^2n(r)}j^2(r)\,dV.$$ (15)

By means of the Maxwell equation $\nabla \times B = 4\pi j/e$ and Eq. (3) for $E_B$, the total energy (13) becomes

$$E = E_S + \frac{1}{8\pi} \int \left( B^2 + \lambda^2(\nabla \times B)^2 \right) \,dV,$$ (16)

where we have assumed that $n$ is constant in the region where there is current, and the London penetration depth is given by

$$\lambda = \sqrt{\frac{mc^2}{4\pi e^2n}}.$$ (17)

Minimizing the energy in Eq. (16) with respect to $B$ then gives the London equation

$$B + \lambda^2(\nabla \times B) = 0,$$ (18)

in one of its equivalent forms. Notice that this derivation utilizes no quantum concepts and does not contain Planck’s constant. It is thus completely classical. A similar derivation has been published more recently by Badía-Majós.

The conclusion of de Gennes is clearly stated in his 1965 book (emphasis from the original): “The superconductor finds an equilibrium state where the sum of the kinetic and magnetic energies is minimum, and this state, for macroscopic samples, corresponds to the expulsion of magnetic flux.” In spite of this, most textbooks continue to state that “flux expulsion has no classical explanation” as originally stated by Meissner and Ochsenfeld and repeated in the influential monographs by London and Nobel laureate Max von Laue. As one textbook example, Ashcroft and Mermin explain that “perfect conductivity implies a time-independent magnetic field in the interior.”

G. A purely classical derivation from magnetostatics

One might object that the electronic charge $e$ is a microscopic constant, and that the London penetration depth $\lambda$ is in most cases is so small that it seems to belong to the domain of microphysics. So even if quantum concepts do not enter explicitly, the above derivation does have microscopic elements. It is thus of interest that from a purely macroscopic point of view we can identify the kinetic energy of the conduction electrons solely with magnetic energy. The energy that should be minimized would then be Eq. (3)

$$E_B = \frac{1}{8\pi} \int B^2 \,dV.$$ (19)

Unfortunately, this will not provide any information about the currents that are the sources of $B$. However, if we can neglect the contribution from fields at a sufficiently distant surface—i.e., when radiation is negligible—it is possible to rewrite this expression as

$$E_B' = \frac{1}{2e} \int j \cdot A \,dV,$$ (20)

where $B = \nabla \times A$. The idea is to then apply the variational principle to $E_{ja} = 2E_B' - E_B$, i.e., the expression

$$E_{ja} = \int \left( \frac{1}{2e} \cdot A - \frac{1}{8\pi} (\nabla \times A)^2 \right) \,dV.$$ (21)

Here the energy is written in terms of the field $A$ and its source $j$. There are many ways of combining Eqs. (19) and (20) to get an energy expression, but it turns out that Eq. (21) is the one that gives the simplest results.

Starting from Eq. (21) and adding the constraint $\nabla \cdot j = 0$, the integration is split into interior, surface, and exterior regions. The result is a theorem of classical magnetostatics that states, for a system of perfect conductors the magnetic field is zero in the interiors and all current flows on their surfaces (Fiolhais et al.). This theorem is analogous to a similar result in electrostatics for electric fields and charge densities in conductors, sometimes referred to as Thomson’s
IV. CONCLUSIONS

The reader may get the impression from our investigations above that we consider superconductivity to be a classical phenomenon. Nothing could be further from the truth. As implied by the Ginzburg-Landau theory, the BCS theory, and the Josephson effect, the phenomenon is quantum mechanical to a large extent. Since quantum physics must lead to classical physics in some macroscopic limit, it must be possible to derive our classical result from a quantum perspective. Evans and Rickayzen did indeed derive the equivalence of zero resistivity and the Meissner effect quantum mechanically, but did not discuss the classical limit. What we want to correct is the mis-statement that the Meissner effect proves that superconductors are “not just perfect conductors.” According to basic physics and a large number of independent investigators, the specific phenomenon of flux expulsion follows naturally from classical physics and the zero resistance property of the superconductor—they are just perfect conductors.

In conclusion, we have carefully examined the evidence for the oft repeated statements in textbooks that (1) there is no classical diamagnetism and (2) there is no classical explanation of magnetic flux expulsion. We have found that the theoretical arguments for these statements are not rigorous and that, for superconductors, these particular phenomena are in good agreement with the classical physics of ideal conductors.


12We only consider magnetism from moving charged particles, not from dipole densities. We therefore need not distinguish between the B and the H-fields. Gaussian units are used.


22X. Fu and Z. Fu, “Realization of Maxwell’s hypothesis,” e-print arXiv, physics/0311104v2 [physics.gen-ph].


Manometric Flame Capsules. The manometric flame was invented by Rudolph Koenig in 1862 for the analysis of sound. The sound is collected at the tube on the right-hand side of the capsule, and causes a flexible membrane that divides the two sides of the capsule to vibrate. The supply of gas entering from the lower left-hand corner is thus modulated, and the small flame burning at the top of the vertical tube oscillates up and down. The rapid oscillations of the flame are observed in a rotating mirror. The wooden capsule was made by the Macintosh Battery and Optical Company, while the gracefully-shaped cast-iron and brass instrument on the left is unmarked. They are both in the Greenslade Collection. (Notes and photograph by Thomas B. Greenslade, Jr., Kenyon College)